國立嘉義大學 99 學年度

應用數學系碩士班（乙組）招生考試試題

科目：機率統計

說明：(1)本試題有機率、統計二大部分，各佔 50 分。
(2)本試題為計算、證明題，請標明每部分的題號，同時將過程作答在「答案卷」上。
(3)統計部分，計算過程如有需要常態曲線下之面積數值，請參考題目後面之附表。

一、機率部分：(50 分)

1. The density function of X is given by

$$f(x)=\begin{cases}1, \textit{if}\ \ 0\le x\le 1,\\ 0,\ \textit{otherwise.}\end{cases}$$

Find the expected value $E[e^X]=?$ (10%)

2. The joint density for the random variables (X, Y), where X is the unit temperature change and Y is the proportion of spectrum shift that a certain atomic particle produce is

$$f(x,y)=\begin{cases}10xy^2, 0<x<y<1,\\ 0, \textit{elsewhere.}\end{cases}$$

(1) Find the marginal density g(x) and the conditional density $f(y\mid x)$. (10%)
(2) Find the probability that the spectrum shifts more than half of the total observations, given the temperature is increased to 0.25 unit. (5%)

3. Let X be a random variable with moments given by

$$EX^n=\frac{(n+1)!}{2^n},\quad n=1,2,3,\cdots.$$

Find the distribution of X. (10%)

4. Show that $\lim_{n\to\infty} e^{-nt}\sum_{k=1}^{n-1}\frac{(nt)^k}{k!}=1$ for $0<t<1,=\frac{1}{2}$ if $t=1$, and 0 if $t>1$. (15%)

二、統計部分：(50 分)

1. Let $X_1, X_2, \cdots, X_n$ be a random sample from an uniform distribution $U(0,\theta)$. Find the maximum likelihood estimator of the reliability function $R(x)=P(X_1>x)$. Is it consistent for $R(x)=P(X_1>x)$? (15%)

2. Let $\bar{X}$ denote the mean of a random sample of size n from a distribution that has mean μ and variance $\sigma^2=10$. Find n so that the probability is approximately 0.9544 that the random interval $(\bar{X}-\frac{1}{2}, \bar{X}+\frac{1}{2})$ includes μ. (10%)

3. Consider the problem of testing $H_0:\mu=10$ versus $H_1:\mu>10$ with $n=64$, standard deviation $\sigma=2$ and at $\alpha=0.025$ level of significance. (10%)
(1) Find the power of this test at the alternative $\mu_1=11$.
(2) Comment on the power in (1) if $\alpha=0.05$ is used.

4. Given the sample data as follows: (15%)
(x, y)= (-2, 0), (-1, 0), (0, 1), (1, 1), (2, 3).
(1) Use the method of least squares of fit a straight line to these data points.
(2) Find the sum of squares for error and estimate the population variance σ^2=?
(3) Fit a parabola to the data using the model $Y=\beta_0+\beta_1 x+\beta_2 x^2+\varepsilon$. (Hint: You may use matrix operations).

Table: Area $\Phi(x)$ under the standard normal curve to the left of x

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990